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PI Controller for Velocity Controller Design based on Lumped Parameter Estimation: Simulation and Experiment

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Abstract: *DC motor is a device that converts electrical energy into mechanical energy. Nowadays, Permanent Magnet DC (PMDC) motors are used in a variety of settings, including residences, commercial buildings, and the manufacturing industry. PMDC motors are currently being used in many applications ranging from everyday tasks to industrial tasks such as industrial machines, automobiles, and robotic applications. In this paper, we present the parameter estimation method of a low-cost PMDC motor using the Extended Kalman Filter (EKF). Each low-cost PMDC motor may have different dynamic and electrical properties such as back electromotive force, torque constant, rotor resistance, and friction coefficients. However, these parameters of the low-cost PMDC motor are not provided in the datasheet by its manufacturer. The main goal of this paper is to estimate the lumped parameters of PMDC motor used in differential-drive mobile robot and design controller for velocity control. SIMULINK software is used to simulate the estimation of the lumped parameters with EKF and then we implement it in real experiment estimation of the lumped parameters. The proportional-integral (PI) controller has been chosen for our feedback control system. Finally, we have found the simulation and real experimental results of three lumped parameters in a brief time. In the simulation results, the estimation values start to converge to true values in only 0.5 seconds, so the simulation and real experimental results confirm that the control is and well-perform.*

Keywords: Parameter estimation, PMDC mtor, PI controller, EKF, Velocity control

1. INTRODUCTION

Nowadays, PMDC motors are very often used as actuators in electromechanical systems in industry and engineering. Because of their low price, compact size, continuous control feature, low voltage, or human safety, PMDC motors are commonly used to control the speed and position of household appliances, portable electronic tools, mobile robots, industrial machines such as printers, wipers, door openers in automobiles, and robot manipulators [1,2]. Due to these advantages, the velocity and position control of PMDC motors have received a lot of study in the literature. Most of the authors contribute models and proposed controllers. Then they make simulations and experiments with hardware to make the realization a reality.

Most manufacturers who sell PMDC motors on the market, particularly low-cost units, do not provide all the dynamic and electrical parameters. These parameters, such as voltage and torque constants or rotor friction coefficients, have a large tolerance. The information provided is basic, such as voltage, power, maximum speed, operating current, etc. It is difficult if we want to do precise control of the mentioned motor and need an expensive testing apparatus, and many testing cycles. So, a quick and effective system identification approach should be proposed. Many research works tried to identify these unknown properties such as the works shown in [3].

Although most industrial control systems depend on PI controllers, most of these applications are nonlinear (temperature control), and PI tuning for nonlinear systems is difficult [4]. On the other hand, Fuzzy PI controllers can be used for nonlinear systems, but they require good knowledge of the system for tuning. The estimation of motor parameters is also essential for condition monitoring, fault diagnosis, etc. However, the identification of non-linear dynamics is very complex, and the design of a controller requires accurate parameters to model the motor shown in [5]. Researchers have attempted different methods of parameter identification for various types of DC

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motors. Different approaches are widely developed for parameter estimation of DC motors [6-10].

In this paper, the DC motor parameters estimation is considered nonlinear. When the speed of the motor changes, it also changes the current and resistance of the rotor as well as the friction constant. Friction is a nonlinear phenomenon that has been studied and modeled differently [11].

The main contribution of this work was improved parameter identification of the PMDC motor by using EKF on the velocity model, which is derived in terms of two lumped parameters. Also, by using the root locus techniques, which provide the whole process of PD position feedback control architecture with acceleration and velocity feed forward as compensation for dynamic reference [12]. So we chose EKF for our research because of its remarkable ability to deal with nonlinear systems and its viability for implementation. We also proposed parameter identification of the PMDC motor by using EKF on the velocity model, which is derived in terms of three lumped parameters, we also provided the full process of PI controller design for velocity feed-forward as compensation for dynamic reference.

This paper is organized as follows: In section 2, showed the model PMDC motor, Extended Kalman Filter Algorithms and velocity using PI controller. EKF implementation, simulation results, implement EKF to real experiment and velocity tracking using PI controller are given in section 3. Finally, the concluding remarks are presented in section 4.

2. METHODOLOGY

2.1 Model PMDC motor

A PMDC motor is a component that consists of an electrical part and a mechanical part. The following PMDC motor model and controller design are derived from work as seen in [13].

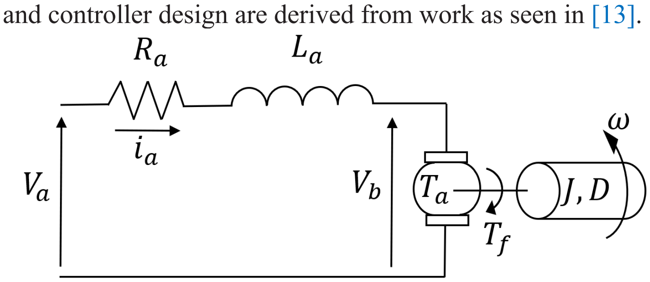

Fig. 1. The schematic of PMDC motor

For the electrical parts from Fig. 1 both the back emf and the electromagnetic torque depend proportionally on speed and current:

$$v_b(t) = K_b\dot{\theta}(t) = K_b\omega(t) \quad \text{(Eq. 1)}$$

$$T_a = K_t i_a(t)\,. \quad \text{(Eq. 2)}$$

$$v_a = L_a\frac{di_a(t)}{dt} + R_a i_a + v_b\,. \quad \text{(Eq. 3)}$$

The inductance L_a in the armature circuit is assumed small and will be neglected in this paper; The equation (Eq. 3) becomes:

$$v_a = R_a i_a + v_b \quad \text{(Eq. 4)}$$

where:

$v_b(t)$ = voltage at terminal conductor of motor (V)
K_b = back emf constant (V/rad. s^{-1})
$\dot{\theta} = \omega$ = angular velocity of motor (rad/s)
T_a = rotor torques (Nm)
K_t = motor torque
i_a = the current draw by the motor (A)
$v_a(t)$ = input voltage from the power source (V)
L_a = inductance in the armature circuit (H)
R_a = the internal resistance of the armature (Ω)

For mechanical part, we have the motor torque equation:

$$T_a = T_f + J\dot{\omega}(t) \quad \text{(Eq. 5)}$$

T_f is the torque that develops from Coulomb frictional torque T_c, and coefficient viscous friction D. T_f is defined as:

$$T_f = T_c sign[\omega(t)] + D\omega(t)\,. \quad \text{(Eq. 6)}$$

where:

T_c = coulomb friction torques (Nm)
T_f = torque of coulomb friction and viscous friction (Nm)
D = coefficient viscous friction (Nm/rad. s^{-1})
J = moment of inertia of the motor (kgm^2)

Substituting (Eq. 6) into (Eq. 5), we obtain

$$J\dot{\omega}(t) + T_c sign[\omega(t)] + D\omega(t) = T_a\,. \quad \text{(Eq. 7)}$$

Using (Eq. 1), (Eq. 2), and (Eq. 4), (Eq. 7) results in

$$\dot{\omega}(t) = -\left(\frac{R_a D + K_t K_b}{R_a J}\right)\omega(t) + \frac{K_t}{R_a J}v_a - \frac{T_c}{J}sign\big(\omega(t)\big). \quad \text{(Eq. 8)}$$

where:

$a = \left(\frac{R_a D + K_t K_b}{R_a J}\right)$ = lumped parameter a (1/s)
$b = \frac{K_t}{R_a J}$ = lumped parameter b (rad/ s^2/ v)
$c = \frac{T_c}{J}$ = lumped parameter c (Nm/ kg.m^2)

2.2 Extended Kalman filter Algorithms

The Extended Kalman Filter is a recursive algorithm which is used for calculating the optimal estimate of $\hat{x}$ state x of the discrete stochastic nonlinear system as below.

$$x_{k+1} = f_d(x_k, u_k) + v_k,$$
$$y_{k+1} = h_d(x_{k+1}, u_{k+1}) + w_{k+1}, \quad \text{(Eq. 9)}$$

where:

x_k = state vector
u_k = input system
y_k = observation vector
f_d = nonlinear functions of state vector
h_d = nonlinear functions of measurement

v_k = process noise vector
w_k = measurement noise vector

Initialize:
$\hat{x}_{0|0}$ = initial state estimation
$P_{0|0}$ = position definite error covariance matrix

Time Update:

$$\hat{x}_{k|k-1} = f_d(\hat{x}_{k-1|k-1}, u_{k-1}),$$
$$P_{k|k-1} = A_{k-1}P_{k-1|k-1}A_{k-1}^T + Q_{k-1}, \qquad \text{(Eq. 10)}$$

Measurement Update:

$$\hat{y}_{K|k-1} = h_d(\hat{x}_{k|k-1}, u_k),$$
$$P_{xy,k|k-1} = P_{k|k-1}C_k^T,$$
$$P_{yy,k|k-1} = C_k P_{k|k-1} C_k^T + R,$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + P_{xy,k|k-1}P_{yy,k|k-1}^{-1}(y_k - \hat{y}_{k|k-1}),$$
$$P_{k|k} = P_{k|k-1} - W_k P_{yy,k|k-1} W_k^T,$$

From linearization of nonlinear function f_d and h_d using Taylor series expansion, we get Jacobian matrix:

$$A_{k-1} = I + T_s \left.\frac{\partial f_d}{\partial x}\right|_{x=\hat{x}_{k-1|k-1}u_{k-1}}, \qquad \text{(Eq. 11)}$$

where:

Q_k = covariance
Q_c = process noise covariance matrix
R = measurement noise covariance matrix
I = identify matrix
A_{k-1} = covariance matrix.

2.3 Velocity control using a PI controller

Then the equation (Eq. 8) can be reduced to

$$\dot{\omega}(t) = -a\omega(t) + bv_a(t) - cign(\omega(t)) \,. \qquad \text{(Eq. 12)}$$

The (Eq. 9) is a nonlinear state equation for the velocity model of a DC motor. The model can be numerically estimated by using EKF. From (Eq. 12), we can design block diagram for velocity model with PI controller as show in Fig. 2.

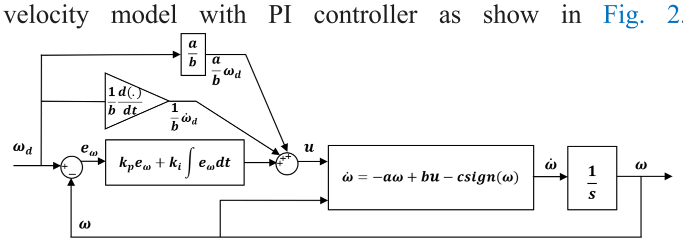

Fig. 2. Velocity model with PI controller

Consider a velocity control architecture with the PI controller as shown in Fig. 2. The governing equation of the control system can be written as:

$$\ddot{e}_\omega + (a + bK_p)\dot{e}_\omega + bK_i e_\omega = 0 \qquad \text{(Eq. 13)}$$

From 2nd Order differential equation in standard form, we have:

$$\ddot{X} + 2\zeta\omega_n\dot{X} + \omega_n^2 X = 0 \qquad \text{(Eq.14)}$$

From (Eq. 13) and (Eq.14), we get:

$$a + bK_p = 2\zeta\omega_n$$
$$bK_i = \omega_n^2$$
$$K_p = \frac{2\zeta\omega_n - a}{b}$$
$$K_i = \frac{\omega_n^2}{b}$$

From the equation above, we want $K_p > 0$. Thus, $2\zeta\omega_n > a$, Then $\zeta\omega_n > \frac{a}{2}$ to ensure stability.

3. RESULTS AND DISCUSSION

3.1 EKF implementation

Here we use the EKF algorithm to estimate parameters *a, b,* and *c* of the velocity model:

Let $x_1 = \omega_{est}, x_2 = a_{est}, x_3 = b_{est}, x_4 = c_{est}$, and $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is the state parameter, input signal $v_a = uk = 10\sin(2\pi + 1.2t)\ (rad), T_s = 0.01s$, $Q_c = 1e - 2 \times diag([1,10,10,10])$ and $R = 0.25$, $v_k: 4 \times 1$ random process noise vector, $w_k: 4 \times 1$ random measurement noise vector. The velocity model (Eq. 12) is rewritten into the stochastic nonlinear system as processing and measuring model below.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_1x_2 + x_3u - x_4 sign(x_1) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \sqrt{Q_c}v(t), \qquad \text{(Eq. 15)}$$

$$y_k = [1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_k + \sqrt{R}\, w(t),$$

Discretize (Eq. 15) for the EKF algorithm, we obtain:

$$x_{k+1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_k + T_s \begin{bmatrix} -x_1x_2 + x_3u_{k-1} - x_4 sign(x_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}_k + \sqrt{T_sQ_c}v(t), \qquad \text{(Eq. 16)}$$

$$y_k = [1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_k + \sqrt{R}\, w(t).$$

The experiment ended after 3 seconds and the values settled to $a = 26.63\ (1/s)$, $b = 17.26\ (rad/s^2/v)$ and $c = 6.776\ (Nm/$

$kg.m^2$). These values are used to compensate for the feed-forward controller. We have shown that EKF could estimate the value of these parameters satisfactorily, with the largest error within 3% from the true value.

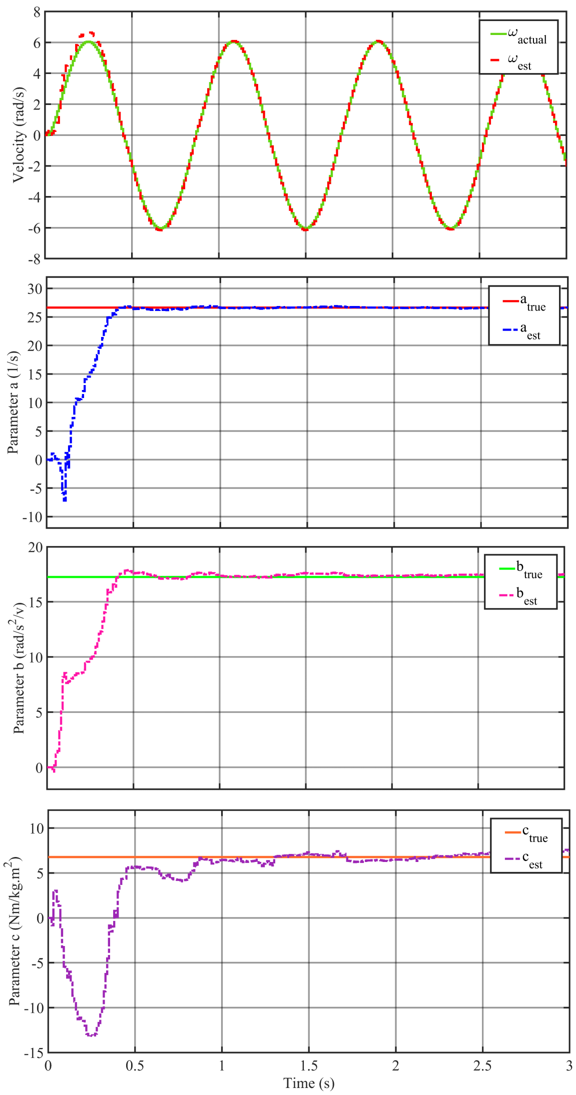

Fig. 3. Simulation results for parameter estimation a, b, and c

As shown in Fig. 3, the angular velocity estimation ω_{est} is start to converge to the angular velocity actual ω_{actual} at time 0.3 seconds, while the error values are quickly approaching to zero value. We compare the angular velocity estimation ω_{est} with angular velocity actual ω_{actual} because we want to show that EKF estimates the values of angular velocity estimation ω_{est} satisfactorily and that it is converge to the angular velocity ω_{actual}. Also, in Fig. 3 is shown the simulation result of parameter estimations a, b, and c with the tuning parameters of EKF. We see that the estimated parameters $x_2 = a_{est}, x_3 = b_{est}, x_4 = c_{est}$, are starting to converge to the true values of 26.63$(1/s)$, 17.26 $(rad/s^2/v)$ and 6.776 $(Nm/kg.m^2)$ at the time of 3 seconds, respectively.

3.2 Experiment lumped parameter estimation

The device, which was built with a low cost PMDC motor, H-bridge driver, and Arduino Due microcontroller, are used for the experiment with hardware testing in MATLAB Simulink. The PMDC motor has gear ratio 19.2, optical encoder 14 ppr and maximum voltage 24 V. The other parameters of the PMDC motor are unknown. The desired dynamic angular velocity is chosen as $uk = 10\sin(2\pi + 1.2t)\ (rad)$ for observation during the experiment. The desired velocity is mathematically calculated from the desired velocity. Compensation for dynamic reference.

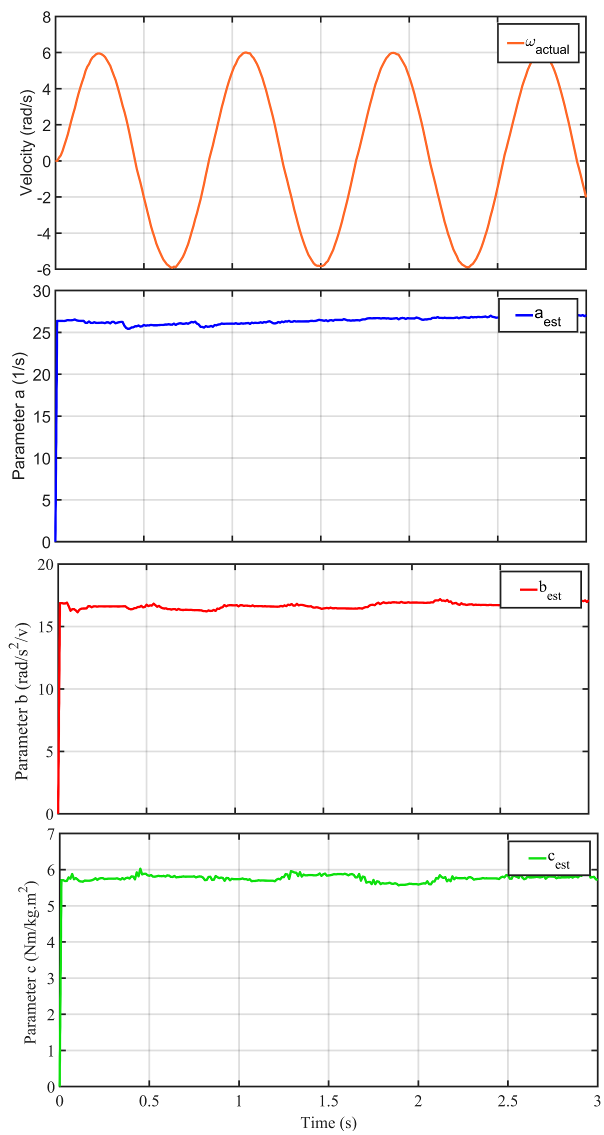

Fig. 4. Experiment results for parameters estimation a,b and c

Fig. 4 shows the experiment ended after 3 seconds and the values settled to $a = 26.63\ (1/s)$, $b = 17.26\ (rad/s^2/v)$ and $c = 6.776\ (Nm/kg.m^2)$. These values are used to compensate for the feed-forward controller. We have shown that EKF could estimate the value of these parameters satisfactorily, with an error maximum of 3% from the actual value.

3.3 Velocity tracking using PI controller

The control architecture in Fig. 2 is used for simulating and experimenting in Simulink for velocity control with PI controller. In this simulation with real experiment, we chose lumped parameters $a = 26.63\ (1/s), b = 17.26\ (rad/s^2/v)$, $\zeta = 1$; $\omega_n = 2 * \pi * 4$; $K_i = \frac{\omega_n^2}{b} = 36.5964$; $K_p = \frac{2\zeta\omega_n - a}{b} = 1.3694$; and input signal $v_a = uk = 10\sin(2\pi + 0.1t)\ (rad)$, $T_s = 0.01\ s$. For the feed-forward ensures that the input to the motor is always regulated based on the motor's dynamic and electrical propertied.

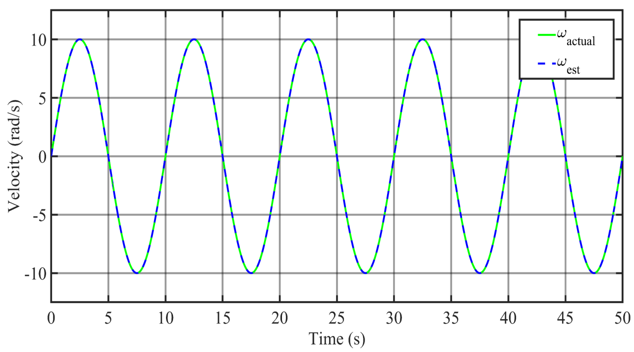

Fig. 5. Simulation results for velocity control architecture with PI controller

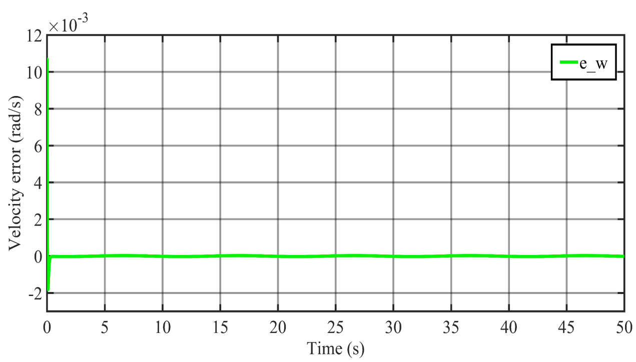

Fig. 6. Simulation results velocity error between estimation and actual with PI controller

Fig. 5 shows the simulation results of velocity control using PI controller. We see that the velocity ω_{est} is starting to converge to the actual velocity ω_{actual} at a time of 1 second while the error values are drastically falling to zero value, and the error between velocity estimate and actual has shown in Fig. 6.

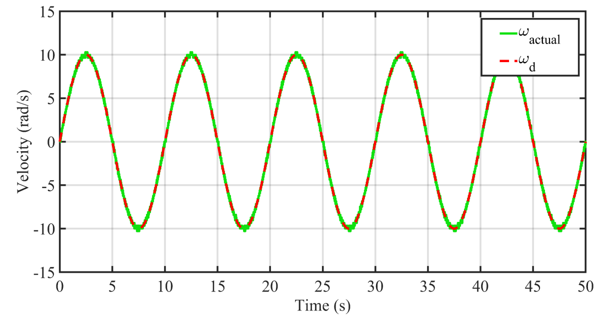

Fig. 7. Experiment results for velocity control with PI controller

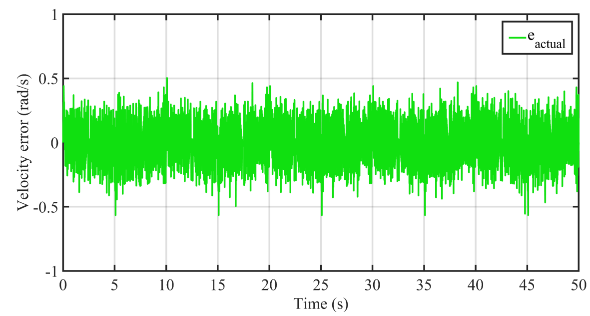

Fig. 8. Experiment results velocity error between estimation and actual with PI controller

4. CONCLUSIONS

In this research, we derive models of a DC motor with three lumped parameters a, b, and c for estimation by using EKF. To simplify the complexity of the parameter estimation, we approximate the models by ignoring the Coulomb frictional effect. EKF is a good method to estimate the lumped parameters a, b, and c of PMDC motors. It has accurately estimated these parameters. These values are used to compensate for the feed-forward controller. We have shown that EKF could satisfactorily estimate the values of these parameters with a maximum error of 3% from the actual value. Then, we use the three estimated parameters to design a PI controller for velocity control. In the parameter estimation, a dynamic reference is used to capture all possible physical properties of the motor. The real experiment was conducted. The experiment results show that the velocity error between estimation and actual with the PI controller is within 0.5 (rad/s) from actual value. For future study, we will apply that work with simulation and real experiments with an adaptive PI controller for velocity control design.

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