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Pose Estimation for Differential Drive Mobile Robot Using Multi-Sensor Data Fusion

Boreth Sethy*, Sarot Srang, Daro Van

Department of Industrial and Mechanical Engineering, Institute of Technology of Cambodia, Russian Ferderation Blvd., P.O. Box 86, Phnom Penh, Cambodia

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Abstract: *The robot localization is a crucial task that needs to be solved as a part of the navigation problem for an autonomous robot. In order to estimate the location of a robot in the environment, various sensors are used to extract meaningful information from measurements to acquire knowledge about the robot's environment and motion. Due to the fact that the sensor uncertainty is random, it is impossible to find an accurate pose for the robot by only one senso,r and the accuracy of any sensor is generally related to its price. Sensor fusion technique is a well-known approach to give the best estimate robot location and how certain it is by combining data from two or more inexpensive sensors. In this paper, the estimation of the robot's new pose given the previous pose and error-accumulated odometry is proposed based on the fusion of data from wheel encoders and the Inertial Measurement Unit (IMU) for a differential drive mobile robot. The robot has a very simple driving mechanism that is quite often used in practice, especially for service mobile robots. The required mathematical models of the robot and indoor localization system are derived. The mathematical tool for sensor fusion is the Kalman filter which provides the optimal estimate of the system state, and robot configuration, assuming that the noise from each sensor is zero-mean and Gaussian. The robot was driven in two different cases; circular trajectory and square trajectory to evaluate the performance and consistency of the robot localization. The experimental result shows the effectiveness of the proposed work for the robot's pose estimation.*

Keywords: Mobile robot; Differential Drive; Pose estimation; Kalman Filter

1. INTRODUCTION

The research of the mobile robot has been being developed rapidly in various fields not only from scientific and engineering perspectives but also with expansion to logistics, service, or social robots nowadays. One of its kinds, the differential driven two-wheels chassis concept has been considered as high mobility with good stability. While advanced simulation and computing programs, such as MATLAB/Simulink provides the tools for modeling, simulating, and analyzing multidomain dynamical that also includes the mobile robots. The mathematical models of a mobile robot with different levels of accuracy based on the mathematical and physical relations will be used as a basis for simulation models programing. These will be useful during the robot prototype design process as well as advanced control designed.

Knowledge of the estimation of location (pose estimate) is a key factor in many applications related to the autonomous system in robotics. The main challenge of localization is to find an accurate pose which is not possible using only one sensor due to sensor uncertainty problem (Thrun et al., 2005). Many attempts to solve this problem, such as finding an accurate pose either by developing new sensors to measure the pose accurately or combining signals from several sensors to get better information from different sources which can help to correct estimated pose or by improving fusing algorithms such as Kalman filter (KF), particle filter, etc. (Elmenreich, 2002; Bräunl, 2008). Most of the sensors are imperfect and susceptible to noisy measurements. Fusing data measurements from multiple sensors is an ideal solution up to date to this problem. In this research, an indoor wheeled mobile robot pose has been estimated based on sensor fusion utilizing Kalman Filter (KF) which provides the optimal estimate of the system state, and robot configuration in this study, assuming that the noise is zero-mean and Gaussian. This filter is a recursive algorithm that, at each time step, updates the optimal estimate of the unknown true configuration and the uncertainty associated with that estimate based on the previous estimate and noisy measurement data (Corke, 2017).

The essential aim of this work is to estimate an accurate indoor mobile robot pose from multiple inexpensive inaccurate sensor models. To achieve this goal, the following

* Corresponding author: Boreth Sethy

E-mail: boreth.sethy@gsc.itc.edu.kh; Tel: +855-78-696-063

steps have been done. First of all, kinematic equations describing the wheeled mobile robot (WMR) motion are derived to offer a general framework for simulation analysis and model-based system control design. Secondly, a theoretical location model of the robot from sensors is derived to further refine the measured data. The sensors chosen are the Inertial Measurement Unit (IMU) and Encoder. The measurement from IMU and encoder is recorded every time step during the movement. Finally, the Kalman Filter (KF) is utilized as a sensor fusion technique to fuse those sensors. The technique proves the accuracy of the estimated pose is improved and well-functioned.

2. KINEMATICS MODEL AND CONTROL

2.1 Kinematics of the Differential Drive Mobile Robot

Differential Drive Mobile Robot (DDMR) is a very simple driving mechanism that is quite often used in practice. The robot with this drive usually has one or more castor wheels to support the vehicle and prevent tilting. Fig.1. shows the kinematics of differential drive robot model with one castor wheel in a 2-D Cartesian workspace. Both main wheels are placed on a common axis and the velocity of each wheel is controlled by a separate motor.

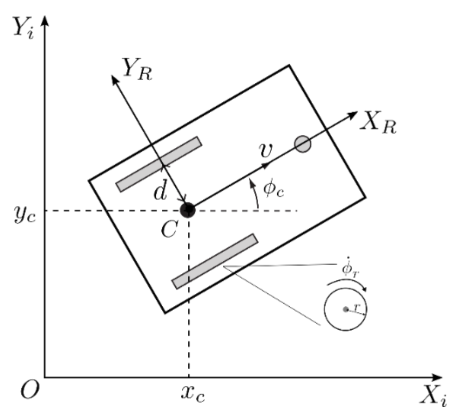

Fig.1. A Differential Drive Mobile Robot

At first, two different coordinate systems are defined, the global coordinate system $\{X_i, O, Y_i\}$ and the robot coordinate system $[X_R, C, Y_R]$ shown in Fig. 1. The global coordinate system is fixed to the Cartesian workspace, and the robot coordinate system is attached to the mobile platform.

For the control purpose, it is often required to define the kinematic model for wheel velocities. Robot kinematic model allows us to represent the robot velocities as a function of the driving wheels velocities along with the geometric parameters of the robot. The distance between the driven wheels is denoted as $2d$. Denote $[x_c, y_c]^T$ as the spatial position of the robot center C, and ϕ_c is the robot orientation angle with respect to point C. The relationship between the velocity in the global coordinate system $[\dot{x}_c, \dot{y}_c, \dot{\phi}_c]^T$ and the velocity $v = [v_c, \omega_c]^T$ in the robot coordinate system can be described as

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi}_c \end{bmatrix}_i = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_c \\ \omega_c \end{bmatrix}. \qquad \text{(Eq. 1)}$$

During the movement of the DDMR, the wheels rotate with angular velocities $\dot{\phi}_r, \dot{\phi}_l$ for the right and left wheel respectively. The linear velocities of each wheel of the robot are related to the wheel angular velocities by:

$$\begin{cases} v_r = r\dot{\phi}_r \\ v_l = r\dot{\phi}_l \end{cases} \qquad \text{(Eq. 2)}$$

Suppose that the robot coordinate system is aligned such that the robot moves forward along X_R. Neither wheel can contribute to sideways motion in the robot coordinate system, and so $\dot{y}_R$ is always zero. Therefore, the velocity of the DDMR in the robot coordinate system is the average of the linear velocities of the two wheels:

$$v_c = \frac{v_r + v_l}{2} = r\frac{\dot{\phi}_r + \dot{\phi}_l}{2} \qquad \text{(Eq. 3)}$$

and angular velocity of the DDMR is:

$$\dot{\phi}_k = \omega_c = \frac{v_r - v_l}{2d} = r\frac{\dot{\phi}_r - \dot{\phi}_l}{2d} \qquad \text{(Eq. 4)}$$

Thus,

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix}_R = \begin{bmatrix} r/2 & r/2 \\ r/2d & -r/2d \end{bmatrix} \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix} \qquad \text{(Eq. 5)}$$

Eq. 1 and Eq. 5 define robot's velocity according to point C for the global coordinate system, and represent the forward kinematic model of DDMR:

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi}_c \end{bmatrix}_i = \frac{r}{2} \begin{bmatrix} \cos\phi & \cos\phi \\ \sin\phi & \sin\phi \\ 1/d & -1/d \end{bmatrix} \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix} \qquad \text{(Eq. 6)}$$

where a positive angular velocity of each wheel corresponds to forward motion at that wheel.

2.2 Tracking Control

The purpose of DDMR path tracking controller is to find a control law input $[v_c, \omega_c]^T$ that the robot can track a desired trajectory $[x_d(t), y_d(t), \phi_d(t)]^T$ in the global coordinate system. The tracking error in the robot coordinate system is defined as $\left[e_x(t), e_y(t), e_\phi(t)\right]_R^T$. The relationship between the tracking errors in the global and robot coordinate systems can be obtained by geometrical projection transformation as:

$$\begin{bmatrix} e_x \\ e_y \\ e_\phi \end{bmatrix}_R = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x_c \\ y_d - y_c \\ \phi_d - \phi_c \end{bmatrix} \quad \text{(Eq. 7)}$$

The control problem will be to determine a control rule for the DDMR, which can estimate $[v_c, \omega_c]^T$ that make the system asymptotically stable. The tracking control law of a typical backstepping technique is given as:

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} k_1 e_x + v_d \cos e_\phi \\ \omega_d + k_2 v_d e_y + k_3 v_d \sin e_\phi \end{bmatrix} \quad \text{(Eq. 8)}$$

where k_1, k_2, and k_3 are positive constants. In addition, Lyapunov stability analysis is also used to prove the system stability and convergence of tracking errors to zero (Dierks & Jagannathan, 2007; Tsai et al., 2004).

SENSORS FUSION

Sensor fusion is a method used to combine data from multiple sensors that measure the same quantity to estimate a more accurate and reliable. Sensors detect a change in the environment and give a signal proportional to this change, then this signal is handled by signal processing units, finally, a fusion sensor algorithm is applied to give a stable and accurate signal out of noisy signals. There are many algorithms to implement sensor fusion. The most commonly used algorithm is Kalman Filter and its family, (Elmenreich, 2002; Thrun et al., 2005).

3.1 Inertial Measurement Unit (IMU)

Inertial based sensor methods, which commonly called inertial measurement units (IMU), are comprised of sensors such as accelerometers, gyroscopes, and magnetometers. These sensors usually deploy together in robots and navigation systems.

- Accelerometer: it is a sensor that measures linear acceleration. The result produced by an accelerometer for mobile robots has been unsuitable and of poor accuracy due to the fact that they suffer from extensive noise and accumulated drift. This can be compensated by the use of a gyroscope.
- Gyroscope: it measures the angular velocity. Gyroscopes run at a high rate, allowing them to track fast and abrupt movement. However, they suffer from serious drift problems caused by the accumulation of measurement errors over long periods.
- Magnetometer: it is another sensor used to determine the heading angle by sensing the Earth's magnetic field. There is a drawback of using magnetometers for indoor positioning because of the presence of metallic objects in the environment that could influence measurement data collection during operation.

When working with a sensor unit containing an IMU, few reference coordinate systems have to be presented. The three major coordinate systems are illustrated in Fig.2.

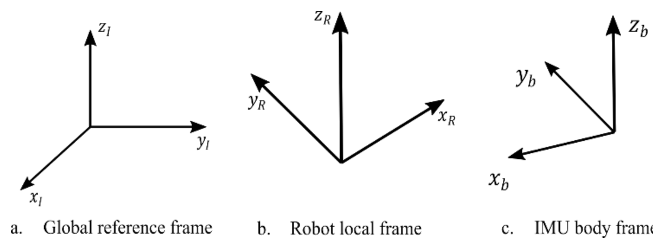

Fig.2. Reference Coordinate System

The sensor chosen for measuring position and orientation was a BNO055 (Townsend, 2019). This is a relatively new chip that is designed for high fidelity navigation applications. This sensor includes three triaxial sensors for measuring acceleration, angular rate, and magnetic fields. The chips have an integrated ARM Cortex-M10 based processor, which performs the sensor fusion and filtering on its own. The maximal update frequency is 100Hz for the Angular Velocity Vector.

3.2 Localization Using Wheel Encoders

The odometer is the measurement of wheel rotation. When odometry measurement is known as a function of time from two wheels on a common axle, the position and orientation of the center of the axle can be determined as a function of time. Generally, the pose of a robot is represented a the vector:

$$p = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix} \quad \text{(Eq. 9)}$$

For a diff-drive robot, the position can be estimated starting from a known position by integrating the movement (summing the incremental travel distance). For a discrete system with fixed sampling interval Δt, the incremental travel distance are (Siegwart et al., 2011):

$$\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta\phi\\ \Delta s\end{bmatrix} = \begin{bmatrix}\Delta s\cos\left(\phi + \frac{\Delta\phi}{2}\right)\\ \Delta s\sin\left(\phi + \frac{\Delta\phi}{2}\right)\\ \frac{\Delta s_r - \Delta s_l}{2d}\\ \frac{\Delta s_r + \Delta s_l}{2}\end{bmatrix} \quad \text{(Eq. 10)}$$

where:
$\Delta s_r; \Delta s_l$: denotes traveled distance for the righ and left wheel respectively,
$\Delta\phi$: denotes angular displacement for the robot.

Then, we further obtain the basic equation for odometric position update at time $k+1$ for differential drive mobile robot:

$$p_{k+1} = \begin{bmatrix}x\\ y\\ \phi\end{bmatrix}_{k+1} = \begin{bmatrix}x_k + \Delta x_k\\ y_k + \Delta y_k\\ \phi_k + \Delta\phi_k\end{bmatrix} \quad \text{(Eq. 11)}$$

3.3 Kalman Filter

Kalman Filter is an optimal estimation algorithm. It is a recursive filter for estimating and filtering a linear Gaussian system. Kalman filter assumes linear state transitions as well as linear measurement (Brown, 1983; Elmenreich, 2002; Grossmann, 1998; Srang, 2014). A linear system is represented in state-space:

$$\begin{aligned}\mathrm{x}_{k+1} &= A\mathrm{x}_k + B\mathrm{u}_k + \mathrm{v}_k\\ \mathrm{z}_{k+1} &= C\mathrm{x}_{k+1} + \mathrm{w}_{k+1}\end{aligned} \quad \text{(Eq. 12)}$$

Table 1. Meaning of matrices in the Eq. 12

Variables	Meaning
x_k	State vector of the dynamical system in $\mathbb{R}^{n_x}$
u_k	Input vector in $\mathbb{R}^{n_u}$
z_k	Measurement of dynamical system in $\mathbb{R}^{n_z}$
A	The state transition matrix in $\mathbb{R}^{n_x \times n_x}$
C	An observation matrix in $\mathbb{R}^{n_z \times n_x}$
B	Control input matrices in $\mathbb{R}^{n_x \times n_u}$
v_k	Independent process noise vector that is assumed to be zero-mean Gaussian with the covariance Q, i.e., $\mathrm{v}_k \sim N(0, Q)$
w_k	Independent measurement noise vector that is assumed to be zero-mean Gaussian with the covariance R, i.e, $\mathrm{w}_k \sim N(0, R)$

The well-known Kalman Filter algorithm for Eq. 12 is given by the steps following:

- Initialization:

Select any initial state estimate and its positive definite error covariance matrix which are denoted as $\hat{x}_{0|0}$ and $P_{0|0}$ respectively.

- Time Update:

$$\begin{aligned}\hat{\mathrm{x}}_{k+1|k} &= A\hat{\mathrm{x}}_{k|k} + B\mathrm{u}_k\\ P_{k+1|k} &= AP_{k|k}A^T + Q\end{aligned} \quad \text{(Eq. 13)}$$

- Measurement Update:

$$\begin{aligned}&\hat{\mathrm{z}}_{k+1|k} = C\hat{\mathrm{x}}_{k+1|k}\,,\\ &P_{\mathrm{xz},k+1|k} = P_{k+1|k}C^T\,,\\ &P_{\mathrm{zz},k+1|k} = CP_{k+1|k}C^T + R\,,\\ &\hat{\mathrm{x}}_{k+1|k+1} = \hat{\mathrm{x}}_{k+1|k} + P_{\mathrm{xz},k+1|k}\,P_{\mathrm{zz},k+1|k}^{-1}\left(\mathrm{z}_{k+1} - \hat{\mathrm{z}}_{k+1|k}\right),\\ &P_{k+1|k+1} = P_{k+1|k} - P_{\mathrm{xz},k+1|k}P_{\mathrm{zz},k|k+1}^{-1}\,P_{\mathrm{xz},k+1|k}^T\,.\end{aligned} \quad \text{(Eq. 14)}$$

In term of the Kalman gain,

$$K_{k+1} = P_{xz,k+1|k}\,P_{zz,k+1|k}^{-1} \quad \text{(Eq. 15)}$$

The measurement update can be rewritten as:

$$\begin{aligned}\hat{\mathrm{x}}_{k+1|k+1} &= \hat{\mathrm{x}}_{k+1|k} + K_{k+1}\left(\mathrm{z}_{k+1} - C\hat{\mathrm{x}}_{k+1|k}\right),\\ P_{k+1|k+1} &= P_{k+1|k} - K_{k+1}P_{\mathrm{zz},k+1|k}K_{k+1}^T.\end{aligned} \quad \text{(Eq. 16)}$$

We use Kalmn filter to estimate the yaw angle ϕ_k of the DDMR, and we use the gyroscope's angular velocity denoted by ω_k as the system input to make the prediction. The angle change $\Delta\phi_k$ obtaited by the encoder is added to the previous estimate as the observed value. The sensor fusion model based on Kalman filter is shown in Fig.3. Then, in the state equation, the state transition matrix $A = 1$, input $\mathrm{u} = \omega_k$, input matrix B is the time difference Δt, and $C = 1$.

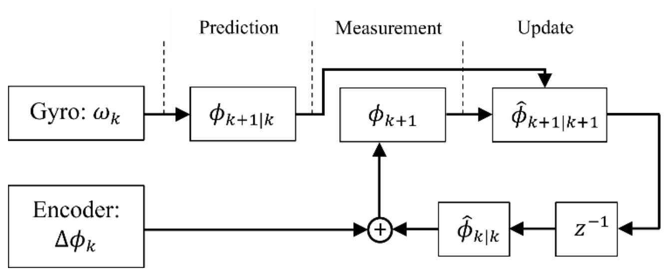

Fig.3. Sensor fusion model

The process noise covariance due to inertial sensor is given as Q, and the measurement noise covariance from encoder model is R. Although the covariance matrices are supposed to reflect the statistics of the noise, the true statistic of the noise is not known. Thus, Q and R are usually used as tuning parameters that we can adjust to get the prospective performance.

3. RESULTS AND DISSCUSION

In this work, the experiments were performed to test the proposed localization method by combining two independent sensors. For comparison purposes, the full system fusing IMU measurement and wheel encoders are compared to the reduced system only wheel encoders. A laptop is fixed to the robot and collected all the needed measurements. To estimate the position, the mobile robot moves as drawing circles and squares in the interest space.

Table 2. Trajectory equations of robot moving

Name of Trajectory	Trajectory Equations
Circular	$x = r\cos(\omega_r t), y = 1 - r\sin(\omega_r t),$ $\phi = 0$, where $r = 1.5m, \omega_r = 0.1rad/s.$
Square	$x = d * abs(\sin(\omega t)) * \sin(\omega t),$ $y = d * abs(\cos(\omega t)) * \cos(\omega t),$ $\phi = 0$, where $d = 1.5m, \omega = 0.1rad/s.$

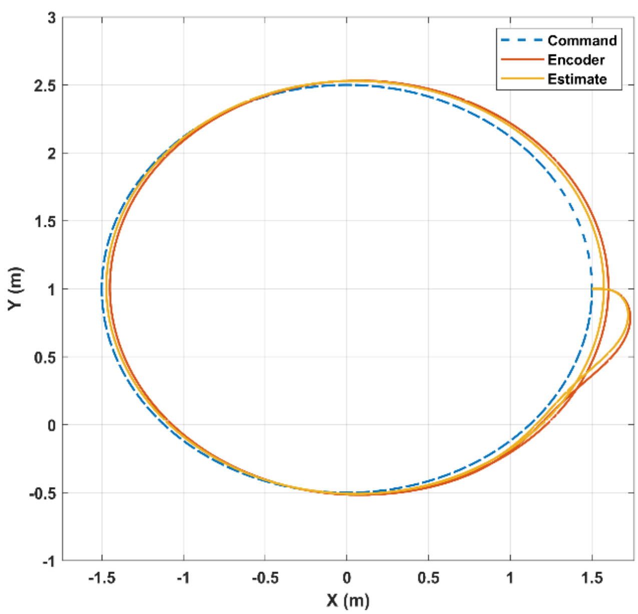

Fig.4. Circular trajectory tracking

The following Figs .4-7 show the experimental results in both case of robot moving for $t = 120$ seconds and for 10ms of sampling time. Although difference between estimated positions were at the small rate, the positioning using only encoder has shown negative bias due to the slips between wheels and ground, and unbounded position error due to the calculated yaw angle changes faster than the true yaw angle. In the experiment results, the proposal algorithm provides reliable position with acceptable level compared to the method that only used encoder.

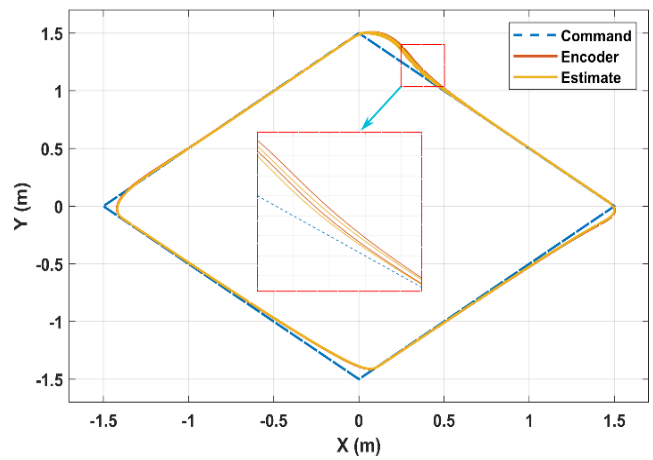

Fig.5. Square trajectory tracking

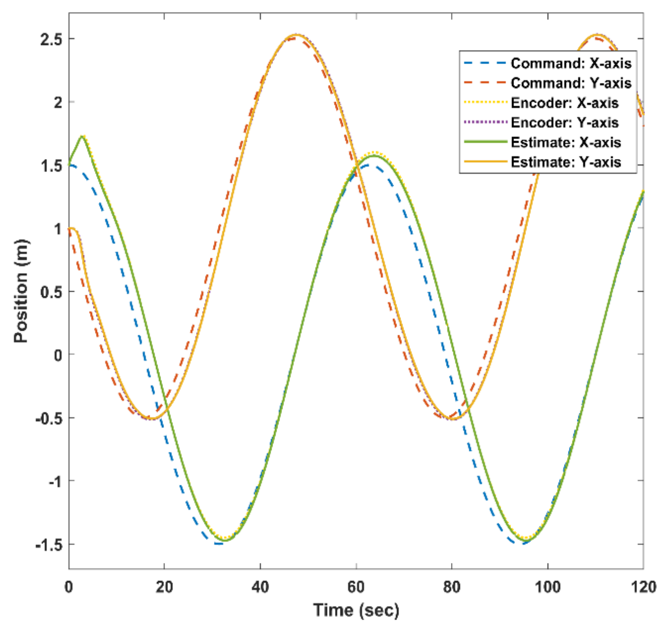

Fig.6. Circular motion trajectories with respect to time

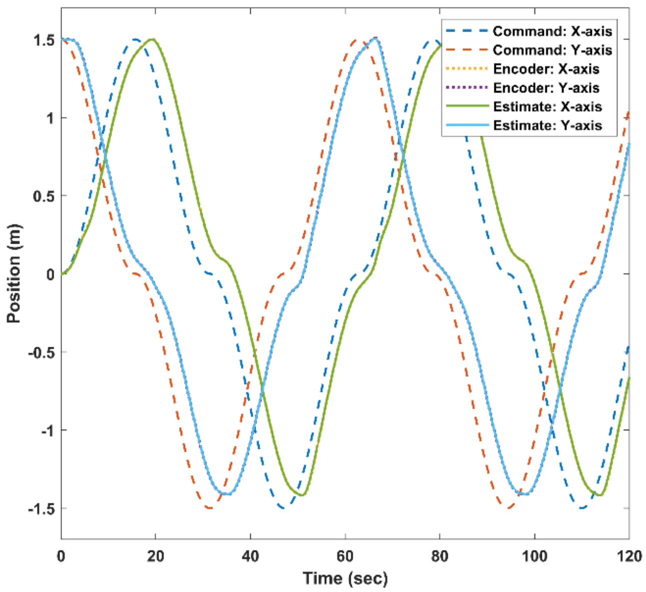

Fig.7. Square motion trajectories with respect to time

4. CONCLUSIONS

In conclusion, we have discussed position estimation of mobile robot in indoor localization. Using the Kalman filter method to fuse two different sensors can guarantee robustness in position displacement and reduce the accumulated errors for a differential drive mobile robot. The aim to design an accurate localization system for this robot was made using a sensor fusion from IMU and wheel encoders. The proposed method compensates for the yaw angle errors that generate position errors in wheel encoders measurement. Therefore, it has smaller position errors and no drift over time.

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